ALTERNATIVE ALGORITHMS

H. Burnett
H. Forbes
March 13, 2013
SCENARIO

A teacher (after having taught the concept of subtraction of two digit numerals) gave the following computation to his class.

\[
\begin{array}{c}
6 & 4 \\
- & 2 & 8 \\
\hline \\
\end{array}
\]
A little fellow in the class gave his solution as follows:

\[
\begin{array}{cc}
6 & 4 \\
\hline
-2 & 8 \\
\hline
-4 \\
\hline
4 & 0 \\
\hline
3 & 6
\end{array}
\]

Was his solution correct or incorrect?
The solution was perfectly correct, the computation however was not done the TRADITIONAL way. It was very obvious that the little fellow was probably mathematically advanced for his class. He actually came up with an algorithm out of the norm or one that his teacher never even thought about!
In the book, *Learning Mathematics in Elementary and Middle Schools*, Cathcart, W.G; Pothier, Y.M. and Vance, J.H. (1994), suggested the following:

1. There is NO ONE CORRECT ALGORITHM. We sometimes alter our normal routine for getting ready for school or work in the mornings, similarly, variations in computational algorithms can be made.
2. It is a common but somewhat narrow view that computation must produce an answer via the traditional algorithm.

3. Different computational algorithms are used in different parts of the world, even by different cultures within the same country. For this reason and additionally because of the mobile nature of the society in which we live, mathematics teachers should be familiar with some of the more common alternative algorithms.
WHAT THEN IS AN ALGORITHM?

• Simply put, an **algorithm** is a step-by-step procedure for producing an answer.

• An **alternative algorithm** then is simply a different way, a non-traditional way or procedure for arriving at the solution for a computation. (remember the little fellow in our opening scenario).
The following reasons propounded by Cathcart (1994) as to why children should explore different algorithms are noteworthy:

1. Alternative algorithms may serve as reinforcement and enrichment.
2. Alternative algorithms provide variety in the mathematics classes.
3. Alternative algorithms may help students develop more flexible mathematical thinking and “number sense.”
4. Awareness of different algorithms demonstrates the fact that algorithms are inventions and can change. This needs to be communicated to students so that they will not develop a belief that there is only one way to perform a mathematical computation.

5. Children can and should be allowed to create and use their own algorithms.
Some alternative algorithms used in computation

• Alternative Addition Algorithms
• Partial Sums:

\[
\begin{array}{c}
2 8 \\
+ 3 4 \\
\hline
1 2 \\
\end{array}
\quad \begin{array}{c}
2 8 \\
+ 3 4 \\
\hline
5 0 \\
\end{array}
\begin{array}{c}
5 0 \\
\hline
6 2 \\
\end{array}
\]
LEFT TO RIGHT

2 8
+ 3 4
15 2
6
62
HALF SPACE NOTATION

\[
\begin{array}{c}
2 & 8 \\
\hline
+ & 3 & 4 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
\phantom{1} \\
2 & \_ & 8 \\
\hline
\phantom{1} \\
3 & \_ & 1 & 4 & 2 \\
\hline
\phantom{1} \\
6 & 2 \\
\end{array}
\]
ALTERNATIVE SUBTRACTION ALGORITHM

• Left to Right

```
  6 3
- 2 4
  4 9
  3
  3 9
```
• Equal addition

\[
\begin{align*}
6 & \quad 1 \quad 3 \\
-  \quad 32 & \quad 4 \\
\hline
\phantom{-} \quad 3 & \quad 9
\end{align*}
\]
LOW STRESS

\[
\begin{array}{cc}
6 & 3 \\
- & 2 4 \\
\hline
5 & 1 3 \\
- & 2 4 \\
\hline
3 & 9 \\
\end{array}
\]
ALTERNATIVE MULTIPLICATION ALGORITHM

• Partial Products:

\[
\begin{array}{c}
3 & 6 \\
X & 2 & 5 \\
6 & 0 & 0 \\
1 & 2 & 0 \\
1 & 5 & 0 \\
\hline
3 & 0 \\
9 & 0 & 0 \\
\end{array}
\quad \begin{array}{c}
3 & 6 \\
X & 2 & 5 \\
3 & 0 \\
1 & 5 & 0 \\
1 & 2 & 0 \\
\hline
6 & 0 & 0 \\
9 & 0 & 0 \\
\end{array}
\]
LATTICE METHOD
Early one spring this Russian peasant was pacing off his field to determine how many cabbage plants he needed for transplanting. Just as he finished, the overseer came riding on his big white horse and demanded, “How many cabbage plants do you need for this spring?”

“We-l-l-l sir, I’m not sure. I know I’ve got room for 25 rows and I can put 36 in each row.”
• “How many is that?” demanded the overseer. “I-I-I’m-m-m not sure, sir”

• “I’m going down the road to see the next farmer. If you don’t know how many plants you need by the time I get back, I’ll have your head.”

• The poor peasant sat down to see if he could figure out how many cabbage plants he needed.
He wrote

Well I could double the 36. That would give me 72. No, I need a lot more than that. Since I doubled 36, maybe I better halve 25.

Let’s see, that is 12 (he didn’t know what to do with remainders)

Perhaps I should do this again. He continued the process until he got to 1 on the left hand side.

“U-m-m-m, I’m sure I need more than 576 Plants. Oh yes that astrologer friend of mine told me that numbers associated with even numbers are unlucky.”
• I’ll strike out the 12 and 6 rows, I can’t see that that helps me. I wonder What would happen if I added this side (pointing to the right-hand column) ? “

He adds 36, 288, 576 to get 900. Just
Then the overseer comes riding back on his big

White horse.

25  36

12  72

6  144

3  288

1  576

900
• Ask children to do examples on their own to tell how they know the answers are correct.

• Children could be challenged with the query, “I wonder if there are any numbers for which the Russian peasant method doesn’t work?”, they will receive considerable self-motivating and self-checking practice.
ALTERNATE DIVISION ALGORITHM

• The Iron Method
• This algorithm relies on a type of compensation property and uses the subtraction or measurement approach to division.


Think $100 - 8$

92) 4765

(100×40) - 4000

765

(8×40) + 320

1085

(100×10) - 1000

85

(8×10) + 80

165

(100×1) - 100

65

(8×1) + 8

73
CONCLUSION

• References


• The greatest invention is still to be made, who knows! One of your students or may be your own child may just come up with the “world’s best algorithm!”

• Continue to empower your students!

    HAPPY

    ALGORITHM!